Keynote Paper

RECENT ADVANCES IN UNDERSTANDING THE RESPONSE OF COMPOSITE MATERIALS AT HIGHER RATES OF STRAIN

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Abstract It is well known that during dynamic deformation or in the event when the deformation is imparted from outside at a very high rate, the stress experienced by the entire structure is not equal at any instance of time. In other words, stress has to travel through the body at a certain velocity usually specific to the body itself. Like quasi-static situation, stresses can not be determined from the sequence of equilibrium states that can be treated by well-known equations of mechanics of materials. Therefore, a separate treatment of the subject based on the propagation of waves is necessary. The wave mechanics when coupled with thermodynamic changes occurring in the body during high rates of loading, makes the analytical and experimental determination of stresses and strain extremely rigorous, and deserves special attention. Over the years, beginning from the early 20th century, researchers have dealt with this issue. Experimental devices were introduced and various analytical formulations were established. While it was progressing well with metals and alloys, complications arose when attempts were made to apply these formulations to composite materials. This paper takes a systematic look at the various approaches undertaken so far and describes state of the art techniques to acquire response of composite materials under high strain rate loading.

Keywords: *Split Hopkinson Pressure Bar ,High Strain Rate, Elastic Wave.*

INTRODUCTION

Composite materials are being widely used as structural components in both civil and military applications. In many of these applications the load is applied dynamically, resulting in the development of high rates of strain and stress. The ability to predict failure of these structures under dynamic loading is becoming increasingly important.

In dynamic events such as impact phenomena, disturbances in the form of stress waves emanate from the point of impact and propagate through the structure. Propagation of these waves through composites will be significantly different from what is usually found with homogeneous materials [1]. Because of the complexity introduced by the inertia forces, such as, variation of properties with the rate of loading, damage initiation and growth during the loading process, etc., the principles of dynamic elasticity have not been well understood. To study the complex phenomena associated with the stress waves, a clear distinction must be made between material response and structural response. On this latter point we note that the response of a structure depends on its geometry, the point of application of the load, and the way in which the material comprising the structure responds. In order to find the individual material response, we must separate it from the overall response of the structure. Experiments must thus be designed or models developed which make this separation possible.

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One of the most widely used test methods for evaluating high strain rate (HSR) effects in materials is the Split Hopkinson Pressure Bar (SHPB) technique. Experimental work on the HSR response of composites involving the use of the SHPB method has been reported in the literature abundantly. Various loading configurations have been used, including compression [2-4], tension [5-7] and shear [8-9]. Additional modifications such as quartz-crystal-embedded Hopkinson bar used by Togami et al. [10], which measures the large-amplitude pulse to evaluate the performance of accelerometers, have extended the capabilities of the SHPB. Recently, Mahfuz et al. [11] further enhanced its application by using a polycarbonate transmitter bar with a steel incident bar to extract the HSR response of soft materials like low-density foam. Although SHPB is a well established and widely used technique for determining HSR behavior of isotropic materials, care must be taken when it is used to characterize the composite materials.

An extensive research has been conducted on Hopkinson bar to verify the validity of basic assumptions used in test procedure in order to provide higher precision to SHPB experiments. The specimen geometry [12-18], dispersion corrections [19-25], radial inertia and frictions [14], validity of testing brittle materials [26] and low impedance materials [27-31] have been thoroughly investigated. HSR inter laminar shear and tensile [32-38] behaviors have also been explored using this technique. However, all these studies deal with the global response of composites. No work has yet been performed to see the individual response of constituent materials using any of the HSR test methods discussed above. Individual response of the constituent material is important for an accurate analysis of the failure modes of composites during dynamic loading.

When used for testing fiber-reinforced polymer composites, the validity of the SHPB method needs to be re-examined carefully because many problems have been identified [39] due to service conditions [40-41] or due to accidental impact [42-43]. In fact, relationships are still undefined between the impact failure strength, failure deformation, absorbed fracture energy, failure mechanism, and material parameters that are known to affect composite behavior [44]. Parameters such as deformation speed [45-46], fiber and matrix type, fiber volume fraction, fiber orientation [47], curing conditions [48] and interfacial bonding [49] influence the composite strength and it is rather difficult to predict the impact strength of composites at HSR loading simply by using the traditional SHPB method. The regular SHPB equations do not consider these parameters or the individual material responses to explain the high strain rate nature of composites. Global deformation of the composite specimen is only considered regardless of the constituent material's behavior at HSR loading.

On the other hand, an impressive number of theories [50-71] have been proposed to explain the elasto-dynamics of composite materials. But all these models and theories predict the overall response of composites during dynamic loading and failures in fibers and matrix are coupled by a single composite failure criterion. These models and theories, although capable of producing important information, are not suitable for the study of complex failure phenomena caused by stress waves inside the composite structures. Recently, novel approaches including new set of mathematical formulations have been developed [72] to extract the individual response of fiber and matrix in a cylindrical composite specimen at HSR loading.

As stated earlier, following the original introduction by Hopkinson [73] and an extensive critical study by Davies [74], Kolsky [1] developed the present form of the SHPB. In the analysis of SHPB measurements, the time duration for direct interpretation of incident, reflected and transmitted pulse from strain gage readings is usually possible for only up to the time of one round-trip wave reflection in the bars [75]. The lengths of the bars and the positions of strain gages are designed such that the incident pulse and the first reflected pulse in each bar can be recorded separately during this period. Duration of these waves also becomes critical in determining the attainment of equilibrium in the specimen. If the superposition of stress pulses travelling in opposite direction takes place, complications arise in strain measurements and make the direct inference of individual pulse difficult. As a result, measurements outside this time window are usually discarded, and the corresponding portion of materials response remains unanalyzed. However, there are cases in which the extended time history of mechanical properties is needed. For instance, when testing low-impedance materials, such as polymeric foams, the desired maximum strain is usually large [76]. Therefore, longer test duration, longer measurements are needed.

It is apparent from the above discussion that the SHPB experiments has been modified substantially over the years with the type of materials to be tested and with the type of response to be acquired from the tests. The following will be an overview of the SHPB experiments and formulations as they have been developed and used to analyze the HSR response of composite materials.

TECHNIQUES FOR MEASURING HSR RESPONSE OF SOFT MATERIALS

Typically, a Split Hopkinson Pressure Bar (SHPB) is used to acquire the response of materials under high strain rate loading. The technique has been extended to composite materials without much of a change except that one has to be careful with the impedance match between the bars and the specimen-material [77-79]. While there are issues still to be resolved, as to how the constituent matrix and fiber will affect the overall composite response, SHPB is a popular technique to characterize composite specimens. In case of soft materials or foam-core sandwich composites, the scenario is however, quite different. Due to the presence of the core material which is usually soft, the magnitude of strain transmitted to the transmission bar will be significantly small. In a regular SHPB set up with steel or aluminum bars, one cannot obtain accurate strain or stress response with such insignificant signal. Some modification of SHPB system to acquire the response of sandwich specimens is therefore, in order.

One idea is to use low impedance material for both incidence and transmission bars so that one can obtain a measurable pulse at the transmission end. A case in point is the use of viscoelastic bars, which has been tried by many researchers [80-84]. Disadvantage with viscoelastic bars is that because of the wave attenuation and dispersion it requires idealized assumptions, which are not necessarily true [85]. Furthermore, the dynamic mechanical properties are also susceptible to environmental effects such as temperature, moisture level and aging factor [85,86]. Due to these limitations, use of viscoelastic bars has not been pursued extensively. Alternately, attempts have been made to use aluminum alloy bars with smaller and hollow cross-sectional area transmitter bar [85]. The goal was to reduce the ratio of the cross-sectional areas between the bar and the specimen such that the transmitted strain can be increased. The concept is good from a theoretical standpoint, but in practice the hollow transmitter bar requires a fitted aluminum cap at its end to hold the specimen. This introduces impedance mismatch and disturbs the stress wave propagation. Although a pulse-shaper is used to filter out the end-cap influence, the question of impedance mismatch still remains.

A novel approach has been developed recently to test sandwich composites using a SHPB. Since only the transmitter signal is weak, a polycarbonate bar is used to replace the transmitter bar as shown in Fig. 1. Because of its low impedance, the polycarbonate bar boosts the transmitted signal to an extent that the regular SHPB data acquisition system can capture the transmitted pulse even for a very soft material. The low impedance of polycarbonate is due to its low modulus and density. Use of polycarbonate bar introduces a different set of problems since now the two bar materials are different. A substantial modification of the SHPB equations is therefore, necessary to utilize this system for strain measurements. Detailed description of the mathematical modifications is presented below.

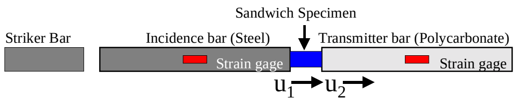

Fig 1. Modified Hopkinson Bar Set Up

MATHEMATICAL MODIFICATIONS

In a regular Split Hopkinson Pressure Bar the nominal strain ε (t) in the specimen is calculated as [1]:

$$\dot{\varepsilon}(t) = -\frac{2C_0}{L}\varepsilon_r(t) \qquad (1)$$

Where L is the original length of the specimen, ε_r (t) is the time-resolved strain of the reflected pulse in the incident bar, and C_0 is the wave velocity through the bar material. Integration of eqn. (1) with respect to time gives the time-resolved axial strain of the specimen. The nominal axial stress, σ, in the specimen is determined as:

$$\sigma(t) = \frac{A_0}{A_s}E_0\varepsilon_t(t) \qquad (2)$$

Where A_s is the cross-sectional area of the specimen, ε_t is the time-resolved axial strain in the transmission bar of cross-sectional area A_0, and E_0 is the Young's modulus of the bar material. The foregoing calculations are based on the assumptions that the specimen undergoes homogeneous deformation and that both the incidence and transmitter bar are made of the same material, and are of same cross-sectional area.

If now a polycarbonate bar replaces the transmitter bar of maraging steel, equation (1) and (2) will no longer be applicable. Materials of the incidence and transmission bars are now different. This difference in materials can be accounted for in the following way.

Let u_1 and u_2, be the specimen-bar interface velocities at the incidence and transmission faces respectively. Now if we assume that the particle velocity is equal to the interface velocity, and that a portion of the incoming strain is reflected at the incidence end, then it can be easily shown that these time-resolved displacements are [72]:

$$u_1(t) = C_i\int_0^t\left[\varepsilon_i(t) - \varepsilon_r(t)\right]dt \quad \text{at } t>0 \qquad (3)$$

$$\text{and} \quad u_2(t) = C_t\int_0^t\varepsilon_t(t)dt \qquad (4)$$

where $\varepsilon_i(t)$ is the incident strain pulse measured by a strain gauge mounted axially on the incident bar surface, C_i and C_t are the incident bar and transmission bar wave velocities respectively. The axial engineering strain of the specimen is therefore,

$$\dot{\varepsilon}(t) = \frac{u_1(t) - u_2(t)}{L}$$

$$\therefore \dot{\varepsilon}(t) = \frac{1}{L}\int\left\{C_i\left[\varepsilon_i(t) - \varepsilon_r(t)\right] - C_t\varepsilon_t(t)\right\}dt \qquad (5)$$

When the specimen is in stress equilibrium, the axial forces acting at the two interfaces are equal, and hence,

$$E_i(\varepsilon_i + \varepsilon_r)A_0 = E_t\varepsilon_t A_0$$

$$\text{or} \quad \varepsilon_t = \frac{E_i}{E_t}(\varepsilon_i + \varepsilon_r) \qquad (6)$$

where E_i and E_t are the Young's modulus of the materials of the incident and transmission bars respectively. Substitution of eqn (6) into eqn (5) gives

$$\varepsilon(t) = \frac{1}{L}\int_0^t\left\{C_i\left[\varepsilon_i(t) - \varepsilon_r(t)\right] - C_t\frac{E_i}{E_t}(\varepsilon_i + \varepsilon_r)\right\}dt$$

or,

$$\varepsilon(t) = \frac{C_i}{L}\left[\left(1 - \frac{C_t}{C_i}\frac{E_i}{E_t}\right)\int_0^t\varepsilon_i(t)dt - \left(1 + \frac{C_t}{C_i}\frac{E_i}{E_t}\right)\int_0^t\varepsilon_r(t)dt\right]$$

$$\therefore \varepsilon(t) = \frac{C_i}{L}\left[(1-K)\int_0^t\varepsilon_i(t)dt - (1+K)\int_0^t\varepsilon_r(t)dt\right] \qquad (7)$$

Where, $K = \frac{C_t}{C_i}\frac{E_i}{E_t}$

Eqn (7) can now be used to calculate nominal axial strain of the specimen from the measured incident and reflected strain pulses using the modified SHPB setup. The average stress in the specimen after the equilibrium is reached, can be found as [72]:

$$\sigma = \frac{P_1(t) + P_2(t)}{2A_0} \tag{8}$$

Where $P_1(t)$ and $P_2(t)$ are the forces acting at interfaces 1 and 2 as defined previously.

Since, $P_1(t) = A_0 E_i (\varepsilon_i + \varepsilon_r)$

and $P_2(t) = A_0 E_t \varepsilon_t(t)$ (9)

Substituting eqns (9) and (6) into (8), one can show that

$$\sigma = \frac{A_0}{A_s} E_t \varepsilon_t(t) \tag{10}$$

If now the strain pulses, namely, $\varepsilon_i(t), \varepsilon_r$ (t), and ε_t (t) are measurable, strain and stress on the sample can be determined from equations (7) and (10) respectively.

Constituent Material's Response

The traditional way to obtain the strain rate, $\dot{\varepsilon}$ for composite specimens, is from the measured values of the strain pulses.

$$\dot{\varepsilon} = \frac{C_0}{L}(\varepsilon_i - \varepsilon_r - \varepsilon_t) \tag{11}$$

Where, C_0 is the longitudinal bar wave velocity, L is the specimen length and ε_i, ε_r, ε_t are the incident, reflected and transmitted strain signals. By integrating equation (11) and assuming the equilibrium state in the specimen, strain of the sample can be calculated using the following equation.

$$\varepsilon = -\frac{2C_0}{L}\int_0^t \varepsilon_r dt \tag{12}$$

On the other hand, from the equilibrium forces at the two interfaces of the specimens, one can calculate the average stress on the sample as:

$$\sigma = \frac{A_0}{A_s} E_0 \varepsilon_t(t) \tag{13}$$

Where, A_0 and A_s are the bar and sample cross-sectional areas respectively, and E_0 is the bar modulus. Stress vs. strain of the sample at the high strain rate loading can therefore be plotted by using the equation (12) and (13).

The regular Hopkinson bar equations give the global response, namely σ(t) and ε (t) of the composite specimen once the equilibrium is reached. Any difference between the responses of matrix and fiber can not be extracted from the regular SHPB equations. In this investigation [72] cylindrical specimen has been designed so that the individual response can be extracted from the regular incidence and transmitted pulse obtained in a SHPB test. The mathematical derivations are as follows:

From the conservation of momentum as applied to the composite specimen sandwiched between the incidence and transmission bars,

$$\frac{d}{dt}(mU_p) = F$$

Where, F = total force on the specimen, m = mass of the specimen and U_p = particle velocity. If F_m And F_f are the loads carried by the matrix and fiber respectively, then

$$F = F_m + F_f$$

It is assumed that both matrix and fiber will move simultaneously so that they will have the same particle velocity, U_p. Since F_m is the load carried by the matrix, a separate momentum equation can be written for the matrix, i.e.,

$$\frac{d}{dt}(m_m U_p) = F_m$$

or, $F_m dt = d(m_m U_p)$

or, $\sigma_m A_m dt = \rho_m A_m dx U_p$

or, $\sigma_m = \rho_m \frac{dx}{dt} U_p$

Thus $\sigma_m = \rho_m C_m U_p$ (14)

since, $\frac{dx}{dt} = C_m$,

Here, $C_m = \sqrt{\frac{E_m}{\rho_m}}$, the wave velocity through the matrix, E_m and ρ_m are the modulus and density of the matrix material and A_m is the cross-sectional area of the matrix in the cylindrical composite specimen. Using the similar approach for the fibers,

$$\sigma_f = \rho_f C_f U_p \tag{15}$$

where, $C_f = \sqrt{\frac{E_f}{\rho_f}}$. It is to be noted here that the modulus, E and density, ρ of both fiber and matrix will be considered constant throughout the loading range. In other words, it is assumed that elastic modulus and the density will remain unchanged with respect to strain rate. From equations (14) and (15) it is established that for a general case,

$$\sigma = \rho C U_p$$

i.e. $\sigma A = F = \rho ACU_p$

Since $F_f = \sigma_f A_f$ and $F_m = \sigma_m A_m$,

$$F = F_m + F_f = \sigma_m A_m + \sigma_f A_f$$

i.e. $F = U_p\left(\rho_f C_f A_f + \rho_m C_m A_m\right)$ (16)

The force, F can now be determined from the strains measured in the incidence bar:

$$F = E_0 A_0 \left(\varepsilon_I + \varepsilon_R\right) \quad (17)$$

where E_0, A_0 are the modulus and cross-sectional area of the incidence bar, and ε_I & ε_R are the incidence and reflected strains respectively measured in the SHPB test. Combining equations (16) and (17),

$$U_p = \frac{E_0 A_0 \left(\varepsilon_I + \varepsilon_R\right)}{\rho_f C_f A_f + \rho_m C_m A_m} = R\left(\varepsilon_I + \varepsilon_R\right) \quad (18)$$

Where, $R = \dfrac{E_0 A_0}{\rho_f C_f A_f + \rho_m C_m A_m}$.

To extract the fiber and matrix strains, equation (15) is used and combined with eqn (18):

$$\sigma_f(t) = \rho_f C_f U_p = \rho_f C_f R\left(\varepsilon_I + \varepsilon_R\right)$$

$$i.e., \varepsilon_f(t) = \frac{1}{E_f}\rho_f C_f R\left(\varepsilon_I + \varepsilon_R\right)$$

Since $E_f = \rho_f C_f^2$

thus, $\varepsilon_f(t) = \dfrac{R}{C_f}\left(\varepsilon_I + \varepsilon_R\right)$ (19)

and similarly for matrix,

$$\varepsilon_m(t) = \frac{R}{C_m}\left(\varepsilon_I + \varepsilon_R\right) \quad (20)$$

Once the equilibrium is reached it can be shown that $\varepsilon_I + \varepsilon_R = \varepsilon_T$ and in that case,

the individual responses are as follows:

the fiber strain is: $\varepsilon_f(t) = \dfrac{R}{C_f}\varepsilon_T(t)$ (21a)

and the matrix strain is: $\varepsilon_m(t) = \dfrac{R}{C_m}\varepsilon_T(t)$ (21b)

Similarly the stress components are:

$$\sigma_f(t) = \rho_f C_f R \varepsilon_T(t)$$

$$\text{and } \sigma_m(t) = \rho_m C_m R \varepsilon_T(t) \quad (22)$$

The set of equations in (21) and (22) can now be used to extract the individual responses of the fiber and matrix from a regular SHPB test.

SEPARATION OF ELASTIC WAVES IN THE SHPB

As stated earlier a method for the analysis of elastic waves in SHPB for unlimited time duration is required to capture the response for a considerable period of time. A method has been developed [75] which allows the separation of component waves traveling in opposite directions in each bar using the strain history measured at one point on the bar and a known end condition for it.

Wave motion in slender cylindrical bar be described by the one-dimensional wave equation

$$\frac{\partial^2 u}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 u}{\partial t^2} \quad (23)$$

The general solution to (23) consists of two arbitrary functions that represent the wave forms traveling in the positive and negative x-directions, *i.e.,*

$$u(x,t) = u_1(t - \frac{x}{c}) + u_2(t + \frac{x}{c}) \quad (24)$$

The longitudinal strain may be expressed in a similar form

$$\varepsilon(x,t) = \varepsilon_1(t - \frac{x}{c}) + \varepsilon_2(t + \frac{x}{c}) \quad (25)$$

where $\varepsilon(x, t) = \dfrac{\partial u(x,t)}{\partial x}$ and functions ε_1 and ε_2 are related to u_1 and u_2 by $c\,\varepsilon_1(\xi) = -\dfrac{du_1(\xi)}{d\xi}$ and $c\,\varepsilon_2(\eta) = -\dfrac{du_2(\eta)}{d\eta}$, respectively.

The particle velocity can be written in terms of ε_1 and ε_2 as

$$v(x,t) = c[-\varepsilon_1(t - \frac{x}{c}) + \varepsilon_2(t + \frac{x}{c})] \quad (26)$$

where $v(x, t) = \dfrac{\partial u(x,t)}{\partial t}$

In a regular SHPB test, ε_1 and ε_2 are determined through direct measurement of strain histories at two different locations on each of the bars. This approach requires an additional gage to be mounted on each of the bars. In a typical situation of the SHPB test, the left end of the incidence bar is free of traction at all times except for the duration when it is in contact with the striker bar.

For the transmitter bar, the right end of the bar is free of traction at all times until it is arrested by a stopper long after the experiment. This known condition can be used to replace one of the measurements needed in the regular method.

From a Lagrangian diagram as shown in Fig. 2, and utilizing eqn. (25) one can write,

$$\varepsilon_A(t) = \varepsilon_1(t - t_a) + \varepsilon_2(t + t_a) \qquad (27)$$

where $\varepsilon_A(t) = \varepsilon(a, t)$ and $t_a = a/c$. The incidence and transmitter bars can now be considered separately because of the differences in their end conditions.

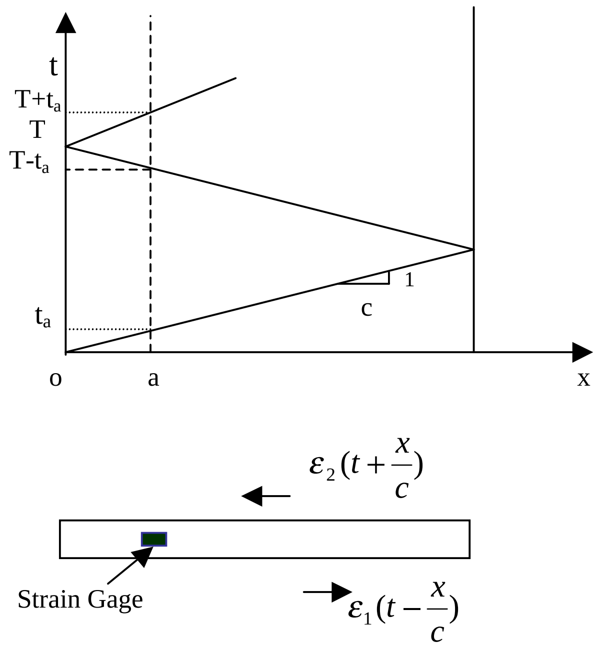

Fig. 2 Lagrangian diagram for longitudinal waves in a cylindrical bar

INCIDENCE BAR

The duration of impact between the striker bar and the incidence bar is $2L_o/c$, where L_o is the length of the striker bar. It is assumed here that the striker bar and the incidence bar have the same impedance. For the incidence pulse and the first reflected pulse from the interface-1 to be recorded separately without superposition at the strain gage at $x = a$, it is required that $L_o < (L - a)$, where L is the length of the incidence bar. Now considering three time intervals, $t < T - t_a$, $T - t_a < t < T + t_a$, and $t > T + t_a$, and invoking the traction free boundary conditions at $x = 0$, and utilizing eqns. (25) and (27) it can be shown that [75]

$$\varepsilon_A(t) = \varepsilon_1(t - t_a) \quad for \quad t < T - t_a \qquad (28)$$

$$\varepsilon_A(t) = \varepsilon_2(t + t_a) \; for \; T - t_a < t < T + t_a \qquad (29)$$

$$\varepsilon_A(t) = -\varepsilon_1(t - t_a) + \varepsilon_2(t + t_a) \; for \; t > T + t_a \qquad (30)$$

These three eqns. can now be combined to yield

$$\varepsilon_1(\xi) = \begin{Bmatrix} \varepsilon_A(\xi + t_a) \; for \xi < T - 2t_a \\ 0 \; for T - 2t_a < \xi < T \\ -\varepsilon_2(\xi) \; for \xi > T \end{Bmatrix} \qquad (31)$$

and

$$\varepsilon_2(\eta) = \begin{Bmatrix} 0 \; for \eta < T \\ \varepsilon_A(\eta - t_a) \; for T < \eta < T + 2t_a \\ \varepsilon_2(\eta - 2t_a) + \varepsilon_A(\eta - t_a) \; for \eta > T + 2t_a \end{Bmatrix} \qquad (32)$$

Where the change in variables are; $(t - t_a) \rightarrow \xi$ and $(t + t_a) \rightarrow \eta$

In a similar fashion, for the transmitter bar with the traction free boundary conditions at $x = L$, it can be shown that [75]

$$\varepsilon_1(\xi) = \begin{Bmatrix} \varepsilon_A(\xi + t_a) \; for \xi < T - 2t_a \\ \varepsilon_1(\xi + 2t_a - T) + \varepsilon_A(\xi + t_a) \; for \xi > T - 2t_a \end{Bmatrix} \qquad (33)$$

$$\varepsilon_2(\eta) = \begin{Bmatrix} 0 \; for \eta < T \\ -\varepsilon_1(\eta - T) \; for \eta > T \end{Bmatrix} \qquad (34)$$

Wave functions can now be determined using eqns. (31) - (32), and (33) - (34).

SUMMARY

A brief overview of the Hopkinson Bar technique in determining the strain and stress pulses is presented. New sets of mathematical formulations have been presented, which with the help of SHPB technique can extract the constituent material's response. It has been observed that the individual responses of matrix and the fibers are significantly different from the global response of the composite even in quasi-static loading conditions. This difference is more pronounced at HSRs. Interfacial failure has also been observed to be directly dependent on the strain rate.

A novel steel-polycarbonate system has been described to test soft materials including sandwich composites at HSR. Related mathematical modifications to regular SHPB equations are also presented. Viscoelastic nature of the polycarbonate bar

(28)
(29)

has been investigated, and its effect is seen to be minimal.

A simple and efficient method for separating component waves travelling in opposite directions in cylindrical elastic bars is presented. The method is based on the one-dimensional wave propagation theory and requires the use of measured strain history at only one location on a bar. The application of this method also requires that the striker bar be shorter than the incidence bar less the distance between the strain gage station and the striker-incidence bar interface. The new method effectively eliminates the limit on the time window for valid data interpretation in the conventional SHPB technique.

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